International Journal of Digital Communication and Analog Signals
Vol. 1: Issue 1
www.journalspub.com

A Genetic Algorithm for Optimization of MSE and Ripples In Linear Phase Low Pass FIR Filter & Also Compare with Cosine Window Techniques

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Abstract

Cosine Window Techniques such as Hamming, Hanning etc. are used to simply design the Finite Impulse Response (FIR) digital Filters. Window function is a scientific function which works on Mathematics by calculating the impulse response of best digital filters. Windowing function is mainly conciliation between the reduction of ripples and Mean Square Error (MSE). But in the proposed research work; Genetic Algorithm (GA) reduces more MSE as well as ripples as compare to Hamming & Hanning window. Bouncing of ripples is known as Gibbs Phenomenon. GA is a Computational search Algorithm which is basically works on the natural selection of genes, chromosomes, crossover and mutation. The proposed paper contains the optimized coefficients, magnitude response, phase response, pole zero map & impulse response of symmetric linear phase low pass FIR Filter.

Keywords*: FIR filter, Mean Square Error (MSE), Hamming, Hanning window, Genetic Algorithm (GA)*

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INTRODUCTION

In the space of Digital Signal Processing (DSP)[1] the design of Filters are basic but a vigorous research topic. Filters are wave shaping devices in time Domain which passes the predefined parts of signal and attenuate the remaining parts. Digital Filters[2,3] are superior to Analog Filters due to its cost effective, reliable & versatile systems. FIR filters are linear phase & stable in nature while the IIR filters are nonlinear & either stable or unstable in nature because IIR filters consist of both poles and zeroes. For the designing of symmetric linear phase FIR filter weights are needed; weights are also called filter coefficients. There are fundamentally three well recognized methods for FIR filter design namely:

1. Frequency sampling method
2. Optimal filter design method
3. Window method

Frequency sampling method is a set of samples which determined from the desired frequency response & are identified as wanted Fourier-Transform coefficients. The filter coefficients are given by t the set of samples of Inverse Discrete Fourier Transform (IDFT) [3]. Frequency Sampling method can be used for any given magnitude response. The frequency response obtained by interpolation is equal to the desired frequency response only at the sampled points. At the other points, there will be a finite error present due to this disadvantage it is not so preferred. Optimal filter design method is basically depending on Chebyshev approximation problem. In this weighted approximation error between the desired filter response & the actual frequency response is spread across the passband and the stopband & the maximum error is minimised [4].

Window methods are basically based on manipulative the impulse response of an ideal digital filter. By window techniques it is simplicity to design the digital FIR filters. Window function is mainly a mathematical function, in this the stop band ripples are reduced and the transition width is also minimises. There are many window techniques like Rectangular, Kaiser, Hamming, Bartlett etc.[5]. In the proposed paper designing of Symmetric Linear phase FIR low pass filter is optimized by Genetic Algorithm (GA) & compared with Hamming & Hanning window techniques.

Genetic Algorithm is an Evolutionary search algorithm[6]. It is based on the Darwin's Theory of "Fittest will survive". GA is used to solve the optimization problem where finding of global minima or maxima enhance the accuracy[7,8]. In the proposed method all unknown FIR filter coefficients are treated as chromosomes; Population of these chromosomes will evolve by different Genetic operations like Reproduction, Crossover and Mutation[9].

GENETIC ALGORITHM

The Genetic Algorithm is an artificial genetic system based on the methodology of Natural selection & Reproduction found in biological system[10]. Initially a set of chromosomes is randomly selected. These chromosomes are encoded as real coded strings called "genotypes". Today, Genetic Algorithm is used to resolve complicated optimization problems[11,12] which is based on "best fittest will survive". The GA works over an iteration process to make the population evolves. Flow chart of GA is as shown below:

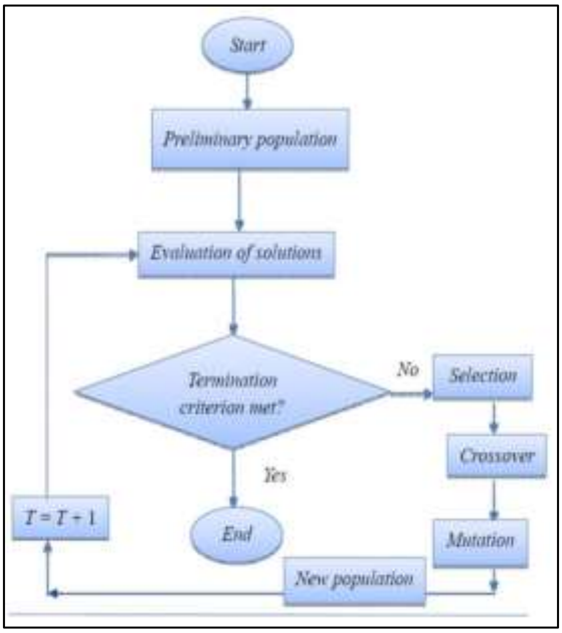

***Fig. 1**: Flow Chart Of Genetic Algorithm.*

Every iteration consists of the following steps:

Initialization: To start the algorithm; selecting some random initial populations i.e. Population Size = 100, 200, 300, 400…………

Evaluation: At the beginning of each generation of algorithms, the fitness of each chromosome must be calculated
$F(X) = f(x)$ for maximization problem
$F(X) = 1/f(x)$ for minimization problem, if $f(x) \neq 0$
$F(X) = 1/(1 + f(x))$, if $f(x) = 0$

Selection: After evaluation, Reproduction is required so for Reproduction individuals are selected. The process of selection is done randomly.

Crossover: It is also called Mating. Generally two chromosomes are called parents. These parents' chromosomes are combined together to form new chromosomes of the child it is also called Offspring[13].

As shown in Figure 2.

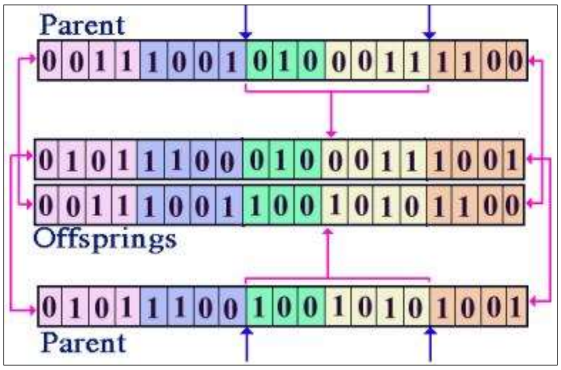

Fig.2: *Formation of Offspring Due To Crossover.*

Parent A and Parent B are two chromosomes which cross each other and form a new child's chromosomes i.e., Offspring.

Mutation: Mutation means that the elements of offspring are modified. It involves a bit flipping i.e., 0 to 1 and vice versa.

DIGITAL FILTER USING WINDOW TECHNIQUE

In the windows method, from the desired frequency response specification $H_d(w)$, corresponding unit sample response $h_d(n)$ is determined using the following relation [4]

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(w)^{ejwn} dw \qquad \text{Eq. (1)}$$

$$H_d(w) = \sum_{n=-\infty}^{\infty} h_d(n)\, e^{-jwn} \qquad \text{Eq. (2)}$$

In general, unit sample $h_d(n)$ obtained from the above relation is infinite in duration, so it must be truncated at some point say $n = M - 1$ to yield an FIR filter of length M (i.e., 0 to M-1). This truncation of $h_d(n)$ to M-1 is same as multiplying $h_d(n)$ by the rectangle window defined as

$$\begin{cases} w(n) = 1 & 0 \le n \le M-1 \\ \quad\;\; = 0 & otherwise \end{cases} \qquad \text{Eq. (3)}$$

Thus the unit sample response of the FIR filter becomes

$$h(n) = h_d(n)w(n) \qquad \text{Eq. (4)}$$

$$h_d(n) = 10 \le n \le M\text{-}1 \qquad \text{Eq. (5)}$$

$$= 0 \quad \text{otherwise}$$

Now, the multiplication of the window function *w(n)* with $h_d(n)$ is equivalent to convolution of $h_d(w)$ with *W(w)*, Where *W(w)* is the frequency domain represent of the window function [3]

$$W(w) = \sum_{n=0}^{M-1} w(n)e^{-jwn} \qquad \text{Eq. (6)}$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(m)\, W(w-m)dw \qquad \text{Eq. (7)}$$

The frequency response can also be obtained using the following relation

$$H(w) = \sum_{n=\infty}^{M-1} h(n)e^{-jwn} \qquad \text{Eq. (8)}$$

But direct truncation of $h_d(n)$ to M terms to obtain *h(n)* leads to the Gibbs phenomenon effect which shows itself as a fixed percentage overshoot and ripples already and afterwards an estimated discontinuity in the frequency response payable to the non-uniform meeting of the Fourier series at a discontinuity. The several effects of windowing the Fourier coefficients of the filter on the result of the frequency response of the filter are as follows

1. The width of the transition bands depends on the width of the main lobe of the frequency response of the window function, w(n) i.e. *W(w).*
2. As M (the length of the window function) increase, the main lobe width of *W*(w) is reduced which reduces the width of the transition band, but this also introduces more ripple in the frequency response.

Some of the windows commonly used are as follows:

1. Rectangular Window
2. Kaiser Window
3. Cosine window function

Cosine Windows

Cosine windows are mainly Hamming, Hanning, Blackman & Bartlett windows.

PROBLEM FORMULATION

For optimizing the filter coefficients a fitness function is needed. A fitness function is basically a squared error between desired value *y[n]* and estimated value $\hat{y}[n]$ as shown in Figure 3. Let the desired value is:

$$y[n]=\sum_{k=0}^{M} b_k x[n-k] \qquad \text{Eq. (9)}$$

$$=b_0 x[n] + \cdots \ldots \ldots \ldots + b_M x[n-M]$$

where '*x*' is the external input and 'y' is the output of FIR filter, b_k is weights that can be optimized. Similarly the calculated value is:

$$\hat{y}[n]=\sum_{k=0}^{M} \hat{b}_k x[n-k] \qquad \text{Eq. (10)}$$

$$= \hat{b}_0 x[n] + \cdots \ldots + \hat{b}_M x[n-M]$$

For applying the proposed genetic operations, an objective function should be defined. So, Minimum Square Error (MSE) is taken as a fitness function, which is given by:

$$\text{MSE}=\sum_{n=0}^{T}(y[n]-\hat{y}[n])^2 \qquad \text{Eq. (11)}$$

$$= \sum_{n=0}^{T} e^2[n]$$

Where *T* is the sampling number. Figure 3.shows the simple estimation block diagram for FIR digital filter by using genetic algorithm.

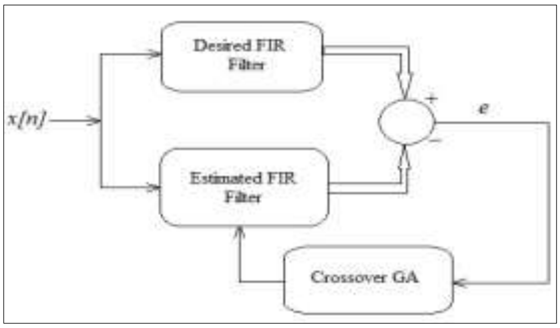

Fig.3: *Block Diagram for FIR Filter Coefficient Estimation Using GA.*

From the Figure 3 *x[n]* is the input for desired and estimated filters. Estimated output is subtracted from desired output gives the Error (e) which is further applied by Genetic approach via crossover and mutation which minimizes the error.

METHODOLOGY

From the MATLAB software the Filter Design Analysis (FDA) toolbox is used for designing of FIR Low Pass filter using Hanning & Hamming window[13]. And for Genetic Algorithm a fitness function from an Equation (11) is created in such a way that the Error between desired value and the calculated value optimize. To run the algorithm the following parameters are set in the optimization toolbox[14].

1. Filter order N = 10
2. Population Size = 500
3. Cut off frequency = 0.5π rad/samples
4. Population Type = Double Vector
5. Creation Function = Constraint dependent
6. Fitness Scaling = Rank
7. Selection = Stochastic uniform
8. Elite count = 2
9. Crossover = 0.8

Population size increases for more accuracy. Elite count is taken as 2 for the best next generation increases; all the parameters are by default in optimization toolbox.

RESULTS & COMPARISON

From the proposed method i.e. Genetic Algorithm the calculated Mean Square Error (MSE) is 0.0175332 after 56 iterations & the MSE of windowing methods are also given below in Table 1.

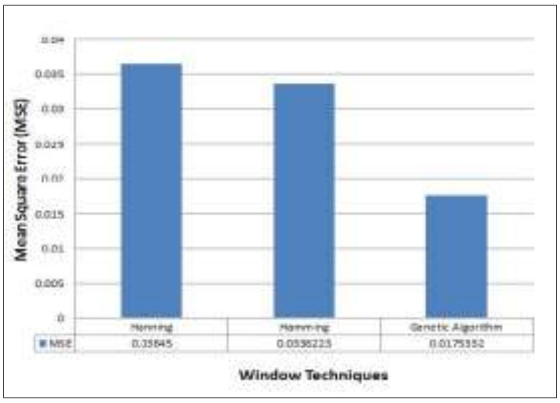

Fig. 4:*.MSE Comparison Between Hanning, Hamming Window & Genetic Algorithm.*

***Table 1**: MSE of Hanning, Hamming Window & Genetic Algorithm.*

Windowing Techniques	**Hanning**	**Hamming**	**Genetic Algorithm**
MSE	0.03645	0.0336223	0.0175332

From Table 1 shows the MSE between the passband and stopband transition width Figure 4 shows the comparison between the MSE of different window Techniques and proposed Genetic Algorithm.

Coefficients

Due to applying Genetic Algorithm coefficients are also optimized of Symmetric linear-phase Low Pass FIR filter which is shown in below Table 2.

***Table 2**: Coefficients of Hanning, Hamming Window & Genetic Algorithm.*

Coefficients	**Genetic Algorithm**	**Hanning window**	**Hamming window**
h(0)	0.0621	0	0.0051
h(1)	0.0028	-0.0000	-0.0000
h(2)	-0.1172	-0.0366	-0.0419
h(3)	0.0029	0.0000	0.0000
h(4)	0.3085	0.2872	0.2885
h(5)	0.4815	0.4987	0.4968
h(6)	0.3085	0.2872	0.2885
h(7)	0.0029	0.0000	0.0000
h(8)	-0.1172	-0.0366	-0.0419
h(9)	0.0028	-0.0000	-0.0000
h(10)	0.0621	0	0.0051

Table 2 shows the optimized coefficients of Genetic Algorithm, Hanning and Hamming window which is symmetric in nature.

Magnitude and Phase

For the designing of FIR low pass filter Hamming and Hanning window is used. The Hamming window is optimized to minimize the maximum (nearest) side lobes, giving it a height of about one-fifth that of hanning window. But after applying GA the transition width is more reduces as compare to Hamming as well as Hanning window & stopband ripples are also reduces gradually[15].

Figure 5 shows the magnitude plot between the normalised frequency & magnitude in dB at cutoff frequency 0.5π rad/samples.
Transition width due to Hamming window = 0.60π rad/sample
Transition width due to Hanning Window = 0.56π rad/sample
Transition width due to Genetic Algorithm = 0.20π rad/sample

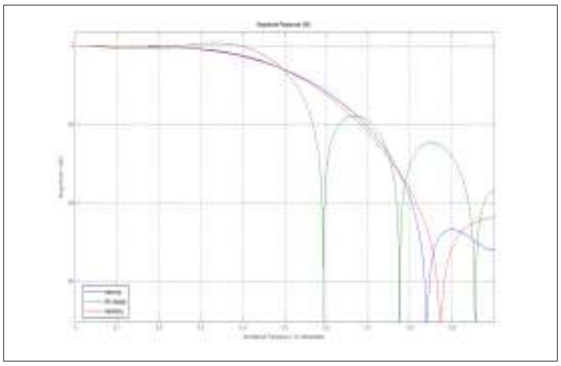

***Fig.5:** Magnitude Response of Hanning, Hamming Window & Genetic Algorithm.*

Phase plot of window function & Genetic Algorithm is given below:

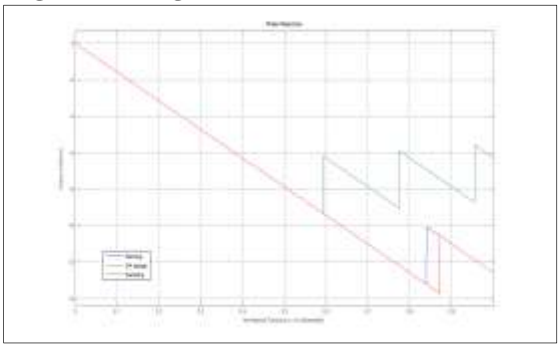

***Fig.6:**Phase Plot of Hanning, Hamming Window and Genetic Algorithm.*

Phase response of FIR low pass filter is Linear in nature as shown in Figure 6. Green line represents the Genetic Algorithm, Blue line shows the Hanning window & Red line represents the Hamming window. The percentage error of transition width is reduced by GA as compare to Hamming & Hanning:
% Error due to GA over Hamming window = {[0.60π - 0.20π]/0.60π}*100 = 66%
% Error due to GA over Hanning window = {[0.56π - 0.20π]/0.56π}*100 = 64%

GA reduces the transition width as compare to Hamming window & Hanning window.

Pole Zero Plot

Pole zero plot of FIR low pass filter with N = 10 order of Hamming, Hanning window and Genetic Algorithm is shown below:

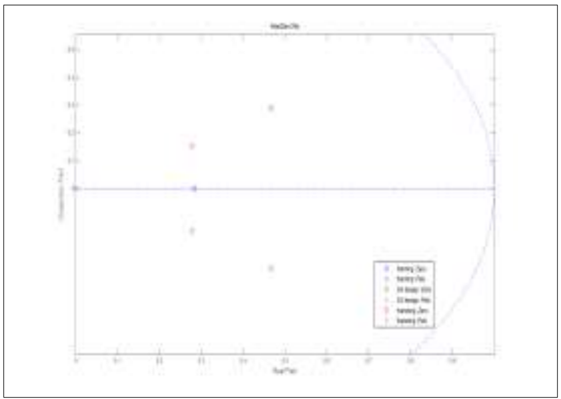

Fig.7: *Pole Zero Map Of Hanning, Hamming Window & Genetic Algorithm.*

The Figure 7 'o' represents the Zeroes and 'x' represents the Poles of FIR Low Pass filter which is symmetric in nature. The stability of filter is depend on mainly Poles in above case the filter is stable in nature[16,17].

Impulse Response

The impulse response of FIR Low Pass filter is shown below with order 10:

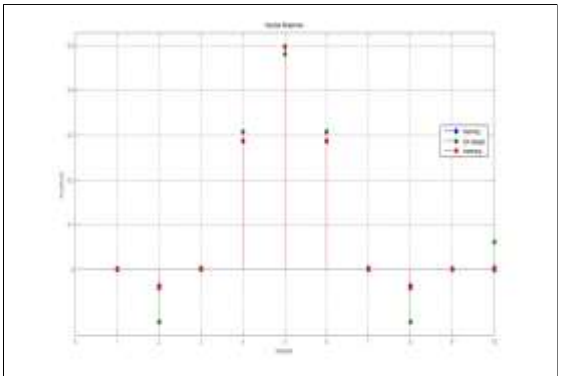

Fig.8: *Impulse Response of Hanning, Hamming Window and Genetic Algorithm.*

Figure 8 represents the impulse response with order 10 of FIR low pass filter after applying the Hanning, Hamming window and Genetic Algorithm where the GA impulse response in slightly improved due to optimization.

CONCLUSION

FIR low pass filter is designed using Genetic Algorithm, Hanning window and Hamming Window as shown in Figure 5 to 8 of Magnitude, Phase , Pole Zero Map and Impulse Response. From the use of Genetic Algorithm; it reduces the Mean Square Error by this the transition region as well as the ripples at passband and stopband also reduces as compare to Hamming window and Hanning window. The % error of reduced Transition band by the GA as compare to window techniques viz. Hanning, Hamming is 64%, 66% respectively which means GA is better than window techniques for designing of linear phase low pass FIR filters.

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